CORRIGENDA ET ADDENDA

AD

ANALYSIN FLUXIONUM.

ANALYSIS FLUXIONUM.

CORRIGENDA. PART I.

PRÆFAT. pag. vii, lin. ult. pro § 131, lege § 26, 27.——p. x, pro § 162, lin. ult. l. § 155 et § 205, not. (*p*)

PARS PRIMA, p. 2, § 5, pro *Daniel*, l. *Jacobus*.——p. 6, not. 2, pro *Institutionis—celebrantur*, l. *Institutis—celebratur*.——p. 9, lin. 1, pro A + B, l. A × B.——p. 10, § 17, pro φωναντα l. φωνεντα.——not. (*d*) lin. ult. pro § 162—164, l. § 154—157.——p. 13, not. lin. 4. a fine, pro *Bezart* l. *Bezout*.——p. 18, § 36, lin. 3, pro *semissis*, l. *semissis* a^2.——p. 25, § 50, lin. 9, pro *For it is*, l. *For in* GEOMETRY *it is*.——p. 31, § 73, lin. 6, pro *nomini*, l. *nomine*.

PARS SECUNDA, p. 35, comma 4, pro *puncti*, l. *puncto*.——p. 39, § 84, lin. 3, pro *latera*, l. *laterum co-efficientia*.——Dele not. (*g*), et transfer ad p. 56, § 124.——p. 42, § 90, com. 1, lin. 6, pro *vel* 2*a*A, l. *vel* 2*b*A.——p. 43, § 92, insere *Cas.* 2. *Fluxio*, &c.——p. 44, § 94.

Cas. 4. Lege, *Fluxio dignitatis cujusvis negativæ* (A^{-m}) *æquatur indici illius dignitatis* (— *m*), *fluxioni lateris* (*a*), *ejúsque co-efficienti* (A^{-m-1}), *in se continuè ductis*. Seu fluxio $A^{-m} = -maA^{-m-1}$.——§ 94, in demonstratione, lin. 3, post $\frac{-a}{A^2}$, insere $= -aA^{-2}$.——et ibidem, lin. ult. post $\frac{-ma}{A^{m+1}}$, insere $= -maA^{-m-1}$. Q. E. D.

P. 45, § 97, lin. ult. ante § 98, insere,—*Cas.* 6. *Cor.*—*Hinc fluxio fractionis* $\frac{A^m}{B^n}$, sive *rectanguli* $A^m B^{-n}$, per *Cas.* 1 et 4, *fiet* $maB^{-n}A^{m-1} - nbA^mB^{-n-1}$, $= \frac{maA^{m-1}}{B^n} - \frac{nbA^m}{B^{n+1}}$. Q. E. D.

 P. 49.

P. 49, § 105, lin. 7, pro *indicet* l. *induit.*——p. 50, lin. 3, à fine, pro *terminis* l. *termini.*——p. 57, lin. 9, insere (sive *cbm* = 1) = $\frac{\dot{x}}{x}$;——p. 57, lin. ult. pro *co-efficienti* l. *co-efficientem.*

Appendix Secunda.—P. 76, not. (*b*), lin. 9, pro *bixapedarum* l. *hexapedarum.*——p. 77, lin. 3, pro *præmissorum* l. *præmissarum.*——p. 84, lin. 16, pro *light* l. *life.*——p. 85, not. lin. 4, à fine, pro *an* l. *our.*——p. 90, lin. 4, pro αωθεν l. ανωθεν.——p. 92, lin. 6, pro *opificis* l. *opificio.*——p. 96, not. lin. 6, pro Νιεν l. Νινευη.——p. 104, not. lin. 10, pro Ηλιος l. Ηελ-ιος.——p. 106, lin. 12, pro *demonstrari* l. *demonstrare.*

ADDENDA.

Pars Prima.—P. 3, § 7. See last page of this, p. 19, § 38, post—*Newtoni discipulis:* nempe, 1. *Robins*, qui anno 1735, in *a Discourse concerning the Nature and Certainty of Sir Isaac Newton's Methods of Fluxions and of Prime and Ultimate Ratios*;—hásce methodos, luculentiùs breviúsque exposuit; magísque ad mentem ipsius inventoris sagacissimi, (meo judicio suffragantibus amicis dignissimis, analystísque simul peritissimis, Episcopo *Horsley* et Barone *Maseres*), quam quilibet ex interpretibus insequentibus; 2. *Colson*, anno insequente 1736, in *the Method of Fluxions and Infinite Series*, &c. 3. *Maclaurin*, anno 1742, in *A Treatise of Fluxions*; et his omnibus multo recentior, 4. *Le Croix*, anno 1779, in *Traitê du Calcul differentiel*, edentibus analystis summis *La Place* et *Le Gendre*; quorum ex operibus excerpta quædam hanc rem illustrantia proferre non pigebit.

I. *Robins's Account of the Methods of Fluxions, and of Prime and Ultimate Ratios.*

"To avoid the imperfection with which the *Method of Indivisibles* was justly charged, (in which all curves are considered as composed of an infinite number of indivisible straight lines, and curvilinear spaces as composed in the like manner of parallelograms; which being an obscure and indistinct perception, was obnoxious to error;) Sir *Isaac Newton* instituted an *Analysis* for these problems concerning the Tangents of Curve-lines, and the mensuration of Curvilinear Spaces, upon other principles.

"Considering magnitudes, not under the notion of being increased by a repeated *accession* of parts; but as generated by a continued motion or *flux*; he discovered a Method to compare together the *velocities* wherewith homogeneous magnitudes

magnitudes increase; and thereby has taught an *Analysis* free from all obscurity and indistinctness.

" Moreover, to facilitate the demonstrations for these kind of Problems, he invented a *synthetic* form of reasoning from the *prime and ultimate ratios* of the contemporaneous augments or decrements of those magnitudes; which is much more *concise* than the method of demonstrating used by the ancients, yet is equally distinct and conclusive.

" I. The method employed by the ancient Geometers (since commonly called the *Method of Exhaustions)* consists in describing upon the curvilinear space a rectilinear one, which, though not equal to the other, yet might differ from it less than by any the most minute difference whatsoever, that should be proposed; and thereby proving the two spaces they would compare, could be neither greater nor less than each other.

" For example, in order to prove *the equality between the space comprehended within the circumference of a circle; and a triangle whose base should be equal to the circumference of that circle; and its altitude to the semidiameter; Archimedes* takes this method:

" About the circle he circumscribes a polygon; and by multiplying the sides of this polygon, he makes it appear, that there may at length be circumscribed such a one as shall exceed the circle less than by any difference that shall be proposed, how minute soever that difference be.—By this means it was easy to prove that the triangle aforesaid is *not greater* than the circle: for were it greater, how small soever be the excess, it were possible to circumscribe about the circle a polygon less than the triangle; but the circumference of the polygon is greater than the circumference of the circle; therefore the polygon can never be less, but must be always greater than the triangle: (for the polygon is equal to a triangle, whose altitude is the semidiameter of the circle, and base equal to the circumference of the polygon.) It appears therefore impossible for the triangle to be greater than the circle.

" Thus far *Archimedes* makes use of the polygon circumscribing the circle, and no farther: but, inscribing another within the circle, he proves, by a similar process of reasoning, that it is impossible for the triangle to be *less* than the circle: whereby at length it becomes certain, that the triangle is neither *greater* nor *less* than the circle, but *equal* to it. Q. E. D.

" However the triangle may be proved *not to be less* than the circle, by the *circumscribed* polygon also: for were it less, another triangle whose base is greater than its base, and height equal, might be taken which would not be greater than the circle; but a polygon can be circumscribed about the circle, the circumference of which shall exceed the circumference of the circle by less than any line that can be named; consequently by less than the difference between

between the two bases; that is, the circumference of the polygon shall be less than the circumference of the circle; and consequently, the polygon less than the given triangle; therefore it is impossible that this triangle should not exceed the circle, since it is greater than the polygon: consequently, the given triangle cannot be less than the circle.

"Thus the circle and triangle may be proved to be equal by comparing them with *one polygon* only: and Sir *Isaac Newton* has instituted upon this principle, a briefer method of conception and expression for demonstrating this sort of propositions than what was used by the ancients; and his ideas are equally distinct and adequate to the subject with theirs, though more complex. It became the first writers to choose the most simple form of expression and the least compounded ideas possible; but Sir *Isaac Newton* thought he should oblige the mathematicians by using brevity, provided he introduced no mode of conception difficult to be comprehended by those who are not unskilled in the ancient methods of writing.

1. Case. "In this method, any fixed quantity, which some varying quantity, by a continual augmentation or diminution, shall perpetually approach but never pass, is considered as the quantity to which the varying quantity will *at last* or *ultimately* become equal; provided the varying quantity can be made in its approach to the other to differ from it by less than by any quantity how minute soever that can be assigned. Princip. Lib. I. Lem. I.

"Upon this principle, the equality between the forementioned circle and triangle is at once deducible: for since the polygon circumscribing the circle approaches to each according to all the conditions above set down, this polygon is to be considered as ultimately becoming equal to both; and consequently, they must be esteemed equal to each other.—That this is a just conclusion is most evident; for if either of these magnitudes be supposed less than the other, this polygon could not approach to the least within some assignable difference.

1 Definition. "An *ultimate magnitude* therefore may be defined the limit to which *a varying magnitude can approach within any degree of nearness whatever, though it can never be made absolutely equal to it.*

"Thus the foregoing circle is to be called the ultimate magnitude of the polygon *circumscribing* it; because this polygon by increasing the number of its sides can be made to differ from the circle less than by any space that can be proposed, how small soever; and yet the polygon can never become equal to the circle or less. In like manner, the circle will be the *ultimate magnitude* of the polygon *inscribed.*

"Again, the foregoing triangle is the *ultimate magnitude*, of the constructed triangle; because, the new base being always equal to the circumference of the polygon, will constantly be greater than the given base, which is equal to the

 circumference

circumference of the circle only; and yet the new base may be made to approach the given one nearer than by any difference that can be named.

2 Case. Ratios also may so vary, as to be confined after the same manner to some determined limit; and such limit of any ratio is here considered as that, with which the varying ratio will ultimately coincide. Princip. Lib. I. Lem. I.

2 Definition. *If there be two quantities therefore, that are (one or both) continually varying, either by being continually augmented or continually diminished; and if the proportion they bear to each other does by this manner perpetually vary, but in such a manner that it constantly approaches nearer and nearer to some determined proportion; and can also be brought at last in it's approach nearer to this determined proportion than to any other that can be assigned; but can never pass it; this determined proportion is then called the ultimate ratio of these varying quantities.*

" The same ratio may be called sometimes the *prime*, at other times *the ultimate ratio* of the same varying quantities; according as these quantities are considered either under the notion of *vanishing*, or of being produced before the imagination by an uninterrupted motion. The doctrine under examination receives it's name from both these ways of expression.

N. B. " The reader will perceive that I am endeavouring to explain Sir *Isaac Newton's* expressions, *Ratio ultima quantitatum evanescentium.* And I have rendered the Latin participle *evanescens* by the English word " vanishing," and and not by the word " evanescent," which having the form of a noun adjective, does not so certainly imply that motion which ought here to be kept carefully in mind: the quantities under consideration become vanishing, from the time we first ascribe to them this perpetual diminution; that is, from the time they are quantities *going to vanish:* and as during their diminution, they have continually different proportions to each other; so the limiting ratio between them, is not to be called merely *Ratio harum quantitatum evanescentium*, but *ultima ratio.* Princip. p. 37.

" I have attempted to explain, in like manner, the phrase *Ratio prima quantitatum nascentium*; but no English participle occurring to me whereby to render the word *nascens*, I have been obliged to use circumlocution. Under the present conception of the varying quantities, they are to be called *nascentes*, not only at *the very instant* of their first production, but (according to the sense in which such participles are used in common speech) after the same manner as when we say of a body which has lain at rest, that it is *beginning to move*, though it may have been *some little time* in motion: on this account, we must not use the simple expression *Ratio quantitatum nascentium*, but to denote the limiting ratio, we must call it, *Ratio prima quantitatum nascentium.* Princip. ibid.

II. " Upon

II. " Upon these definitions, we may ground the following Propositions :

1 Prop. *When varying magnitudes keep constantly the same proportion to each other, their ultimate magnitudes are in the same proportion.*

" Let A and B be two varying magnitudes, which keep constantly in the same proportion to each other ; and let C be the ultimate magnitude of A, and D the ultimate magnitude of B : I say that C is to D in the same proportion as A to B.

Since A is a varying magnitude continually approaching to C, but can never become equal to it, A may be either always greater or always less than C.

In the first place suppose it *greater :* when A is greater than C, in approaching to C, it is continually diminished ; therefore B keeping always in the same proportion to A, B in approaching to it's limit D, is also continually diminished :

A B
C D
E

Now, if possible, let the ratio of C to D be *greater* than that of A to B : (that is, suppose C to have to some magnitude, E, greater than D, the same proportion as A to B).

Since C is the ultimate magnitude of A in the sense of the preceding definition, A can be made to approach nearer to C than by any difference that can be proposed, but can never become equal to it or less : therefore, since C is to E as A to B, B will always exceed E ; consequently, can never approach to D so near as by the excess E : which is absurd : For, since D is supposed the ultimate magnitude of B, it can be approached by B nearer than by any assignable difference.

After the same manner, neither can the ratio of D to C be *greater* than that of B to A.

In the second place, if the varying magnitude A be *less* than C ; it follows in like manner, that neither the ratio of C to D can be *less* than that of A to B ; nor the ratio of D to C *less* than that of B to A. Q. E. D.

Cor. *The ultimate magnitudes of the same or equal varying magnitudes are equal.*

" Now from this corollary (which evidently follows from the proposition) the forementioned equality between the circle and triangle will immediately appear : for the circle being the ultimate magnitude of the polygon, and the given triangle, the ultimate magnitude of the constructed triangle ; since the polygon and the constructed triangle are equal, by this corollary, the circle and given triangle will be also equal. Q. E. D.

N. B. " If the preceding proposition and it's corollary be admitted as genuine deductions from the [First] definition upon which it is grounded ; this demonstration

demonstration of this equality of the circle and triangle cannot be excepted to: for no objection can lie against the definition itself, as no inference is there deduced, but only the sense explained of the term [*Ultimate magnitude*] defined in it."

2. PROP. "*All the ultimate ratios of the same varying ratio are the same with each other.*

"Suppose the ratio of A to B continually varies by the variation of one or both of the terms A and B: if the ratio of C to D be the ultimate ratio of A to B, and the ratio of E to F be likewise the ultimate ratio of the same; I say, the ratio of C to D is the same with the ratio of E to F.—For, if you deny it, the ratio of E to F differing from the ratio of C to D, the ratio of A to B will either pass that of E to F, or can never approach so near it as to the ratio of C to D: insomuch that the ratio of E to F cannot be the ultimate ratio of A to B, contrary to the hypothesis. Q. E. D.

"The two definitions here set down, together with the general propositions annexed to them, comprehend the whole foundation of this method.

III. "We find in former writers some attempts towards so much of this method as depends upon the first definition.

"*Lucas Valerius*, in a most excellent treatise on *the Center of Gravity* of Solid Bodies, has given a proposition nothing different but in the form of the expression, from that we have subjoined to our first definition: from which he demonstrates with brevity and elegance his propositions concerning the mensuration and center of gravity of the sphere, spheroid, parabolical and hyperbolical conoids. This author writ before the doctrine of *Indivisibles* was proposed to the world.

"And since, *Andrew Tacquet*, in his treatise on the *Cylindrical and Annular Solids*, has made the same proposition, though something differently expressed, the basis of his demonstrations; at the same time very judiciously exposing the inconclusiveness of the reasoning from *indivisibles*.

"However, the consideration of the *limits of varying proportions*, when the quantities themselves expressing those proportions have no limits, (which renders this *Method of prime and ultimate ratios* much more extensive,) we owe entirely to *Sir Isaac Newton*. That this method, as thus compleated, is applicable not only to the subjects treated of by the Ancients in the Method of Exhaustions, but to many others also of the greatest importance, appears from our author's immortal Treatise on the *Mathematical Principles of Natural Philosophy*.

"For it must now be manifest, that *mathematical quantities* become the proper object of this *Doctrine of Fluxions*, whenever they are supposed to increase

by any continued mode of prolongation, dilatation, expansion, or other kind of augmentation; provided such augmentation be directed by some *general rule* whence the measure of the increase of these quantities may *from time to time* be estimated. And when different homogeneous magnitudes increase after this manner together, one may vary *faster* than another. Now the *velocity of increase* in each quantity is the *fluxion* of that quantity. This is the true interpretation of Fluxions; *Incrementorum velocitates*. For this doctrine does not suppose the fluents themselves to have any motion: fluxions are not the velocities with which the fluents or even the increments which these fluents receive are *themselves* moved; but *the degrees* of velocity wherewith those increments are generated—the terms *velocity* and *celerity* are applied in a *figurative* sense, to denote the *degree* [or *rate*] wherewith this augmentation *in every part* proceeds.

"Subjects incapable of *local motion*, such as fluxions themselves, may also have their fluxions. In this we do not ascribe to these fluxions any *actual* motion; (for, to ascribe motion or velocity to what is itself only a velocity would be wholly unintelligible.) The fluxion of another fluxion, is only a *variation* in the velocity which is that fluxion. In short, *light*, *heat*, *sound*, the *motion of bodies*, the *power of gravity*, and whatever else is *capable of variation*, and of having that *variation assigned*, for this reason, comes under the present doctrine: nothing more being understood by the *fluxion* of *any* subject, than *the degree* [or *rate*] *of it's variation*.

"As the *Doctrine of Fluxions* enabled Sir Isaac Newton to investigate the *geometrical* problems, whereby he was conducted in those remote searches into Nature, which have been the subject of universal admiration; so to the *Method of Prime and Ultimate Ratios* is owing the *surprising brevity*, wherewith he has *demonstrated* those discoveries."

I shall offer no apology for the length of this *Analysis*, in which I have endeavoured to bring together into one comprehensive view the scattered parts of that masterly argument, by which *Robins* has explained the leading principles of the Doctrine of *Prime and Ultimate Ratios*, upon which Newton's *Method of Fluxions* is founded; it is far superior indeed to the subsequent explanations of professed commentators; and it is a high gratification to myself to find, that the mode of explanation, which I adopted of the Doctrine of Limits, is precisely the same as *Robins's*; long before I had seen his admirable treatise, which did not fall into my hands until lately, a considerable time after the publication of the *Analysis Fluxionum*. It deserves indeed to be better known and more studied; as containing a full and sufficient refutation of the cavils of gainsayers both ancient and modern, against *the nature and certainty of the Method of Fluxions*.

II. *Colson's*

II. *Colson's Account of the Method of Fluxions*, &c.

III. *Maclaurin's Account of the Method of Fluxions*, &c.

After the end of Maclaurin's Account, &c. p. 23, insert

IV. *La Croix's Account of the Method of Fluxions.*

To the foregoing testimonies of the most eminent *British* mathematicians I am happy to add the following, which reflects high honour on the candor and liberality of a distinguished *French* analyst, *La Croix*, confessing the superiority of the *Method of Fluxions* over the *Differential Calculus*; from a curious and valuable Extract furnished even by the prejudiced MONTHLY REVIEW, 1800. Vol. 31. Append. p. 497.

"*Newton* supposes lines to be generated by the motion of a point; and surfaces, by the motion of a line; and he gave the name of *Fluxions* to the velocities which regulated the motions. These notions, although rigorous, are foreign to Geometry, and their application is difficult. It is true that, by imagining a point which moves on a line, while the line itself is carried forward with an *uniform* velocity, we may represent any curve whatsoever: but the velocity of the describing point being *variable*, at each instant, we can only determine it by recurring to the Method of the Ancients (*Exhaustions*), or to that of *Prime and Ultimate Ratios*.

"It is of this last method that *Newton* almost always avails himself; so that, properly speaking, *Fluxions* were to him only a mean of giving a *sensible existence* to the quantities on which he operated. By the *Method of Prime and Ultimate Ratios* he understood the investigation of the relations of quantities *at the first and last instant of their existence*, when the quantities were generated or vanished together; and he found in the prime ratio of spaces described by the ordinate on the line of the abscissas, and by the describing point of the ordinate (spaces which he called *Moments*), the ratio of the fluxion of the abscissa to that of the ordinate; whence he determined the direction of the tangent. The Calculus was merely that used by *Barrow* in his Method of Tangents, which *Newton* by means of his Formula for the Binomial Theorem, and by his reduction into series, had extended to irrational expressions.

"The advantage of the *Method of Fluxions* over the *Differential Calculus*, in point of *Metaphysique*, consists in this: That, *fluxions* being *finite* quantities, their *moments* are only infinitely small quantities of the *first order*, and their *fluxions are finite*: by these means, the consideration of *infinitely small quantities of superior orders* is excluded."

How was it possible for *Monthly Reviewers*, after reciting this luminous and honourable testimony to the superiority of the Method of Fluxions by the ablest expounders of the Differential Calculus, *La Croix* and his illustrious editors *La Place* and *Le Gendre*, redeeming the character of the *French* analysts, which had been impaired by the aspersions of a *D'Alembert* and of a *La Grange* on the immortal *Newton's* fame; how was it possible, I say, that "their eyes could still be so holden," after adducing this testimony, as still to assert, that "*Newton himself was not perfectly satisfied of the stability of the ground on which he had established his Method of Fluxions!*"—*to wonder*, [*how*] *that* [*after*] *having beyond all controversy obtained* TRUTH, *mathematicians should have been unable to make it* SCIENCE; *for the method was simple and easy in it's application and rigorous in it's conclusions!*"—"*Viewed as a whole it possessed the greatest stability; though it's foundations seen through a mist, seemed uncertain and of discordant and unsuitable materials!*"—"*Had the native insignificancy of the Fluxionary Calculus doomed it to perish, the wit and poignant raillery of Berkeley had perpetuated it's memory, and 'ridiculed it into immortality:' but in spite of these attacks, the English Mathematicians have still persevered in their opinions; deeming it perhaps more meritorious to err with* NEWTON *than to think justly with* OTHER *men*," [i. e. THE MONTHLY REVIEWERS]! pp. 495—499.—And they will still persevere in their attachment to the Father of British Science.

How widely different are the sentiments of the illustrious *La Place*, which they cite in the next page, 500, from his Letter to M. *La Croix*.

"I see, with much pleasure, that you are engaged in a great work on *the Differential and Integral Calculus. The several methods, by being brought together, will throw mutual light on each other. What they contain in common is most generally their true metaphysique: and this is the cause why the metaphysique is almost always the last thing that is discovered.—It is only by reflecting on the route which* OTHERS *have followed, that* WE *are able to generalize methods and to discover their true metaphysics.*"

This indeed is worthy of a mighty master of the Sciences, and a genuine disciple of *Newton*, treading in his steps, and thereby surpassing his teacher.

Page 29, after § 68, and before § 69, insert:

The following masterly explanation of the term *momentum*, as employed by *Newton*, is given by *Robins*, in the *Conclusion* of his excellent tract, p. 75.

"In determining the ultimate ratios between the contemporaneous differences of quantities, it is often previously required to consider each of these differences *apart*, in order to discover *how much* of those differences is necessary for expressing that ultimate ratio. In this case, Sir *Isaac Newton* distinguishes by the name of *momentum*, *so much of any difference as constitutes the term used in expressing this ultimate ratio.*

"Thus,

"Thus, if A and B denote varying quantities, and their contemporaneous increments be represented by *a* and *b*, the rectangle under any given line M and *a* is the contemporaneous increment of the rectangle MA; and A*b* + B*a* + *ab* is the like increment of the rectangle AB.—And here, *the whole* increment M*a* represents the *momentum* of the rectangle MA; but *the part*, A*b* + B*a*, only, (and not the whole increment A*b* + B*a* + *ab*,) is called the *momentum* of the rectangle AB: because *so much only* of this latter increment is required for determining the ultimate ratio of the increment of MA to the increment of AB; this ratio being the same with the ultimate ratio of M*a* to A*b* + B*a*: (for the ultimate ratio of A*b* + B*a* to A*b* + B*a* + *ab* is the ratio of equality. Consequently, the ultimate ratio of M*a* to A*b* + B*a* differs not from the ultimate ratio of M*a* to A*b* + B*a* + *ab*. Q. E. D.

"These *momenta* equally relate to the *decrements* of quantities as to their *increments*; and the ultimate ratio of increments and of decrements *at the same place* is the same. Therefore the *momentum* of any body may be determined, either by considering the increment or the decrement of that quantity; or even by considering *both* together. And in determining the momentum of the rectangle AB, Sir Isaac Newton has taken the last of these methods; because by this means the superfluous rectangle (*ab*) is sooner disengaged from the demonstration.

"Here it must always be remembered, that the only use which ought ever to be made of these *momenta* is to compare them one with another, and for no other purpose than to determine "*the ultimate or prime proportion between the several increments or decrements from whence they are deduced.*" § 66.

"Herein the *Method of Prime and Ultimate Ratios* essentially differs from that of *Indivisibles*; for in that method, these *momenta* are considered absolutely as *parts*, whereof their respective quantities are *actually* composed. But though these *momenta* have no *final magnitude*, (which would be necessary to make their *parts* capable of compounding a whole by accumulation,) yet their *ultimate ratios* are as truly assignable, as the ratios between any quantities whatsoever. Therefore none of the objections made against the doctrine of *Indivisibles* are of the least weight against this method: but while we carefully attend to the observation here laid down, we shall be as secure from error, and the mind will receive as full satisfaction, as in any the most unexceptionable demonstration of Euclid."

P. 82. Insert in the beginning of note (*e*),

The opinion of the Indian *Brahmins* is thus recorded by *Strabo*, B. 15.

Νομιζειν μεν γαρ δη ΤΟΝ ΕΝΘΑΔΕ ΒΙΟΝ *ὡς αν ακμην κυομενον ειναι*, ΤΟΝ ΔΕ ΘΑΝΑΤΟΝ *γενεσιν εις τον οντως βιον, και τον ευδαιμονα τοις φιλοσοφησασι.*

"For

"For they are accustomed to account *the present life here*, as if it were an embryo only; but *death*, a birth into the real life and the happy, reserved for the seekers of wisdom."

The following curious and valuable anecdote, &c.

Pars Prima, p. 3, § 7, after l. 7. Vide quoque § 96 hujus—Insert:

Idque insuper constat, ex ipsius Newtoni testimonio, *Quadrat. Curvar.* sub initio: "*Incidi paulatim annis* 1665 *et* 1666, *in Methodum Fluxionum, quâ hic usus sum in Quadraturâ Curvarum.*"—

CORRIGENDA. PART II.

P. 56, § 122, dele l. 8 et 9, et substitue insequentia:

Eritque *ratio modularis* in quovis systemate logarithmorum ratio ista cujus logarithmus est ipse *modulus*. Hæc autem ratio in omni systemate eadem erit; scilicet ratio seriei infinitæ $1 + \frac{1}{1} + \frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4}$, &c. ad 1, ejúsve reciproca: seu ratio numeri 2.71828, &c ad 1, ejúsve reciproca, nempe, ratio fractionis decimalis 0.367879, &c. ad 1.

N. B. Vulgò fertur, et quibusdam auctoribus etiàm melioris notæ placet, (scilicet, *Montucla*, &c,) logarithmorum systemata *Briggianum* atque *Neperianum* non nisi diversis hyperbolis, et diversis curvis logarithmicis, seu logisticis, designari posse; sed minùs rectè, ut videtur: nam, si unitas designet quadratum constans *hyperbolæ rectangulæ*, (contentum utique asymptotis et ordinatis externis hísce parallelis), areæ inter ordinatas uni ex asymptotis parallelas, alteram asymptotam, ipsámque hyperbolam interclusæ, logarithmorum systema *Neperianum* ritè exponent; sin verò area inter duas ordinatas uni ex asymptotis parallelas quarum major sit decupla minoris, et alteram asymptotam, et hyperbolam ipsam, interclusa per unitatem designetur, seu vocetur unitas, areæ asymptoticæ ejusdem hyperbolæ, quæ anteà systema Neperianum exponebant, nunc æquo jure systema Briggianum exponent.

In *quâvis* autem *curvâ logisticâ*, seu *logarithmicâ*, si subtangens curvæ (quæ per totam curvam est semper ejusdem magnitudinis,) per unitatem designetur, abscissæ axis, seu asymptotæ, inter duas ordinatas interceptæ, logarithmos systematis

ſyſtematis *Neperiani* exponent; ſi verò abſciſſa quævis axis, ſeu aſymptotæ, inter duas ordinatas, quarum major ſit decupla minoris, intercepta, (quæ paritèr per totam curvam erit ſemper ejuſdem magnitudinis,) per unitatem deſignetur, eædem abſciſſæ axis, ſive aſymptotæ, quæ anteà exhibebant logarithmos ſyſtematis Neperiani, nunc exhibebunt logarithmos ſyſtematis Briggiani.

Hinc conſtat, epitheton "*hyperbòlicum,*" logarithmis *Neperianis* vulgò tributum, *Briggianis* aut *alterius cujuſvis ſyſtematis* logarithmis æquè competere. Vide Cl. *Maſeres's Diſſertation on the Nature of Logarithms*, in his *Elements of Plane Trigonometry*.

123. *Fluxiones logarithmorum*, &c.

P. 57, dele totum *Caſ.* 2, et p. 58 totam N. B.; pro quibus ſubſtitue quod hic ſequitur;

Caſ. 2. Fiat $y^x = z$. Erìtque log. $z =$ log. $y \times x$. Sed log. z eſt $= \frac{\dot z}{z}$, per § 127; et log. $y \times x$ eſt $=$ log. $y \times \dot x + x \times \frac{\dot y}{y}$, per § 84. Ergò $\frac{\dot z}{z}$ erit $=$ log. $y \times \dot x + x \times \frac{\dot y}{y}$; et proindè, $\dot z$ erit $=$ log. $y \times \dot x z + xz\frac{\dot y}{y}$ $=$ log. $y \times \dot x y^x + x\dot y y^{x-1}$, reſtituendo ſcilicet y^x pro z; hoc eſt, ***fluxio quantitatis variabilis*** (y^x) ***æquatur duabus quantitatibus, quarum una*** (log. $y \times \dot x y^x$) ***eſt fluxio ipſius quantitatis exponentialis*** (y^x) ***quaſi pro conſtanti habitæ***, ut in Caſ. 1; ***altera autem***, ($x\dot y y^{x-1}$) ***fluxio ejuſdem quantitatis, quaſi indici conſtanti deſignatæ***. Q. E. D.

FINIS.

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Systematis *Neperiani* exponent; si verò abscissa quævis axis, seu asymptoti, inter duas ordinatas, quarum major sit decupla minoris, intercepta, (quæ pariter per totam curvam erit semper ejusdem magnitudinis,) per unitatem designetur, eædem abscissæ axis, sive asymptoti, quæ curva exhibebant logarithmos systematis Neperiani, nunc exhibebunt logarithmos systematis Briggiani.

Hinc constat, epitheton "*hyperbolicum*," logarithmis *Neperianis* vulgò tributum, *Briggianis* aut *alterius cujusvis systematis* logarithmis æquè competere. Vide Cl. *Maseres's Dissertation on the Nature of Logarithms*, in his *Elements of Plane Trigonometry*.

125. *Plurimas logarithmorum*, &c.

P. 57, dele totum *Cas.* 2, et p. 58 totam Q. E. D.; pro quibus substitue quod hic sequitur;

Cas. 2. Fiat $y^x = z$. Eritque log. z = log. $y \times x$. Sed log. z est $= \frac{\dot{z}}{z}$, per § 127; et log. $y \times x$ est = log. $y \times \dot{x} + x \times \frac{\dot{y}}{y}$, per § 84. Ergo $\frac{\dot{z}}{z}$ erit = log. $y \times \dot{x} + x \times \frac{\dot{y}}{y}$; et proinde, $\dot{z}$ erit = log. $y \times z\dot{x} + xz\frac{\dot{y}}{y}$ = log. $y \times \dot{x}y^x + x\dot{y}y^{x-1}$, restituendo scilicet y^x pro z; hoc est, *fluxio quantitatis variabilis* (y^x) *æquatur duabus quantitatibus, quarum una* (log. $y \times \dot{x}y^x$) *est fluxio ipsius quantitatis exponentialis* (y^x) *quasi pro constanti habitæ*, ut in *Cas.* 1; *altera autem*, ($x\dot{y}y^{x-1}$) *fluxio ejusdem quantitatis, quasi indicis constantis designata*. Q. E. D.

FINIS.

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